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Comparative analysis on an exponential form of pulse with an integer and non-integer exponent to use in pulse compression with better resolution in range and velocity

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Abstract
The theme of this paper is to make a fundamental comparative analysis on time-bandwidth product of a short duration pulse, whose amplitude is varied with an exponent as an integer and non-integer. The time-bandwidth product is the most significant factor in pulse compression techniques which is used very often in radar systems for better detection of the target and resolving the ambiguities in the both range and velocity with the help of ambiguity function. In this paper, different exponents have been used and it is observed that the non-integer exponents are giving slightly better quantitative parameters like time-bandwidth product, relative sidelobe level and main lobe widths. To improve these quantitative parameters, phase variations have been incorporated with the differentiated pulses of the original exponential signal. Finally, the modulation has also been applied on the pulses to observe the results in real practical applications. From all these analysis, it is concluded that the differentiated non-integer exponential pulse with bi-polar variations is giving better pulse compression requirements (time-bandwidth product, peak sidelobe level, 3-dB beamwidth and main lobe widths) compared to all other pulse forms. The outputs of the matched filter are also observed for each pulse shape.
Keywords: Radar,pulse compression, ambiguity function, range resolution, velocity resolution,

I. Introduction

Radar systems [1-3] are generally used to track the objects in moving environments. For tracking an object, an electromagnetic wave will be transmitted from a source and the reflected signal from the moving object (target) will be captured by the receiver and signal processing techniques will be used to find the distance at which the target is located from some knownreference called as range. Similarly, the velocity with which that target is moving with relative to thereceiveris called as the velocity of the target.

In order to determine the range of the target, the delay between the transmitted pulse and reflected pulse will be measured and knowing the propagation speed of the electromagnetic wave in the space or different media, the distance can be calculated from the delay between the both the pulses. For practical applications rather than transmitting a single pulse, pulses at some repetitive interval will be transmitted whose period is called as pulse repetitive interval (p.r.i). If the radar systems have to identify different targets at different locations on the same radial line, there has to be some constrain on the duration of the transmitted pulse so that the two nearby targets can be resolved as two distinct targets rather than as a single target. The minimum distance between two nearby targets that can be resolved as distinct targets is called as range resolution and given by

$$R = c\tau/2 \qquad (1)$$

where$R$ is the range resolution,τ is the pulse duration and c is the velocity of electromagnetic wave.

In order to determine the velocity of the object, the relative frequency shift between the transmitted signal and the reflected signal is measured and this Doppler frequency shift will be used to find the radial velocity of the object using

$$f_d = 2v_r/\lambda \qquad (2)$$

where the Doppler frequency shift is $f_d$and v_r is the radial velocity of the target. If two targets are moving with two different velocities which are close by, even then the radar system must identify them as two different targets which is represented as the velocity resolution. In general a radar system has to resolve the targets in both range and velocity without any ambiguities and this puts a constraint on the pulse duration and the peak average power that has to be transmitted in pulse mode. For good resolution in range, short duration pulse is needed while for transmitting power constraint a long duration pulse has to be transmitted. The resolution in frequency can be obtained as the reciprocal of the time duration and if the spectral spread is more, then that pulse is more suitable for the better resolution in frequency domain. This leads to the problem of pulse compression where the duration of the pulse has been fixed at constant level and to look for the other alternatives where by the spectrum can be extended in frequency domain. This is done by modulation and there has been a lot of research in this field which gives better pulse compression techniques [4].

So many techniques for pulse compression have been developed, such as linear frequency modulation (Costas), non-linear frequency modulation, Barker codes, phase modulation (poly phase and bi-phase)[5-7] . Even though there has been lot of research carried out on pulse compression, this paper presents the pulse compression with the basic pulse in exponential formand its variants which are obtained by differentiating the original pulse and multiplication with signumfunction. Finally, multiplying the modified pulses with a sinusoidal to relate with practical case of transmission.

This paper has been organized as follows. The section II gives the problem statement while section III gives the simulation results. In this paper ambiguity function has been analyzed and the results for this function are discussed in section IV and section V gives the conclusion based on the detailed study.

II. Problem statement

In radar systems, the transmitted signal is represented with $x(t)$ and the received signal can be represented as $r(t)$.The received signal can be approximated in terms of transmitted signal with some attenuation, delay and frequency shift in the frequency domain as

$$r(t) = \alpha(t)x(t-T)e^{j2\pi f_d t} \qquad (3)$$

where$\alpha(t)$ can be taken as attenuation of the signal as a function of time. Usually, this can be considered as a constant. Trepresents the delay between the transmitted signal and received signal while the multiplication with the complex exponential represents the variation in the frequency of the received signal due to relative motion between target the radar system.

The time-bandwidth product is the figure of merit for the pulse compression and hence the transmitted pulse frequency spectrum is obtained with the help of fast Fourier transform (FFT) [8].To make a comparative analysis, different pulses have been considered and time-bandwidth products have been tabulated.

In radar systems, to maximize the peak signal to noise ratio a matched filter is used which acts like a correlator and its impulse response can be given as

$$h(t) = r^*(\tau - t) \qquad (4)$$

whereτ is the parameter used for maximizing the output of the filter at predefined time.

The output of the matched filter can be used to find the peak side lobe levels in the autocorrelation function which gives an idea about the range resolution and velocity resolution.PSLis the peak side lobe level, which measures the ratio of maximum side lobe magnitude to the in phase value of the Auto Correlation Function(ACF). This can be calculated as [9]

$$PSL = 20log_{10}\frac{max_{\,1\le l\le N}|C(l)|}{|C(0)|} \qquad (5)$$

N is number of side lobes, C(l) is output of matched filter.

It is often necessary to examine a waveform and understand its resolution and ambiguity in both range and speed domains. The range is measured using delay and speed is measured using the Doppler shift. In order to measure range and speed of an object ambiguity function can be used, that is represented as

$$X(t_d, f_d) = \int_{t=-\infty}^{\infty} x(t)x^*(t-t_d)\, e^{j2\pi f_d t}dt (6)$$

$X(t_d, f_d)$is ambiguity function, t_d and $f_d$are the time Delay and Doppler shift respectively.
In this paper an exponential pulse of the form

$$x(t) = \alpha \cdot e^{f(t)}.rect(t/\tau) \qquad (7)$$

where$f(t)$ is of the form $\beta \cdot t^n$ where β is a constant and exponent n is integer and non-integer as well. Along with this simple pulse, this pulse has been modified with differentiation and multiplication with signum function, and modulating these pulses with a sinusoidal carrier signal. All simulations are carried out on MATLAB [10].

III. Simulations and results

In the first place a simple exponential pulse has been analyzed. Mathematically this pulse can be represented as

$$x_1(t) = \alpha \cdot \left(e^{t^n}.u(-t) + e^{-t^n}.u(t)\right) \qquad (8)$$

Here n, α are the order and amplitude of the signal. Exponential pulses having non-integer order of half and integer order of one, two, three, and five has been analyzed for every case. Fig.1 represents Time domain and frequency spectrum analyses for exponent signal.Frequency spectrum of the exponential pulses has been observed with the Fast Fourier Transform (FFT).

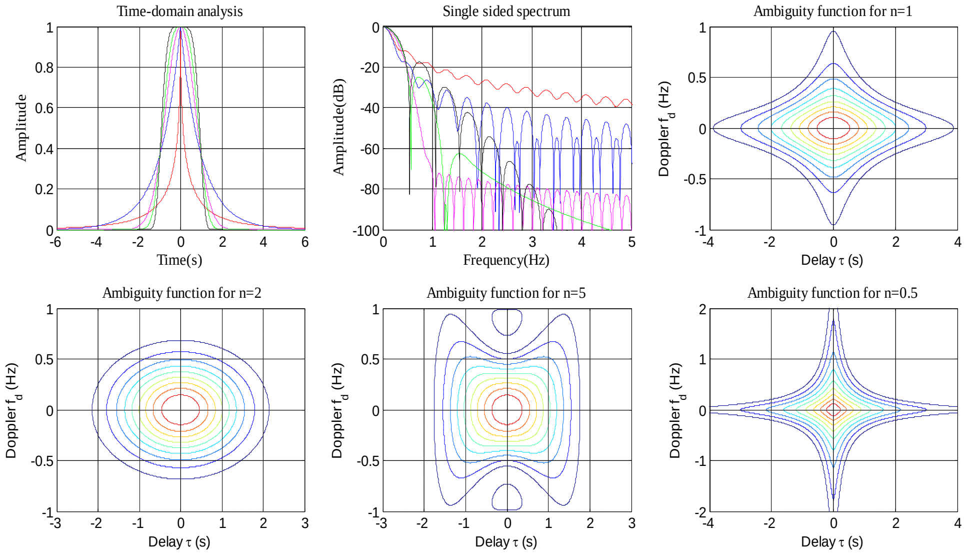

Fig.1 Time domain, frequency domain and ambiguity functions for the exponential signal ($x_1(t)$) with different exponents. (Red trace is for n=0.5, blue trace is for n=1, magenta is for n=2, greenis for n=3 and black is for n=5).

From Fig.1, it is observed that for order two its spectrum is flat in comparison with other orders, but matched filter outputs are not compressed. After observing this, the polarity of the pulse has been changed as positive half and negative half for the entire pulse. This is like multiplying the given pulse with signum function. Mathematically bi-phase exponential signal can be represented as

$$x_2(t) = \alpha \cdot \left(e^{t^n}.u(-t) - e^{-t^n}.u(t)\right) \quad (9)$$

Fig.2 represents the time domain and frequency analysis of bi-phase exponent pulses for different order. The color of the traces has been preserved in all the succeeding figures for comparison.

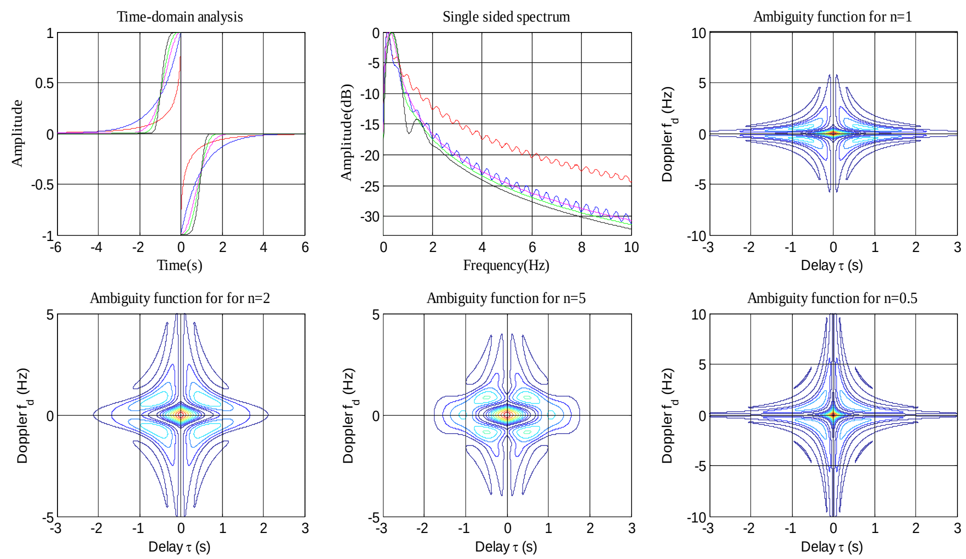

Fig.2 Time domain, frequency domain and ambiguity functions for the exponential signal ($x_2(t)$) with different exponents.

A simple mathematical operation is done on the original exponential pulse by differentiation. Differentiated exponential pulses can be represented as

$$x_3(t) = \alpha \cdot \left(n.t^{n-1}.e^{t^n}.u(-t) - n.t^{n-1}.e^{-t^n}.u(t)\right) \quad (10)$$

Fig.3represents the Time domain and frequency domain analysis for differentiated pulses of different orders. For order two have good frequency spectrum than others.

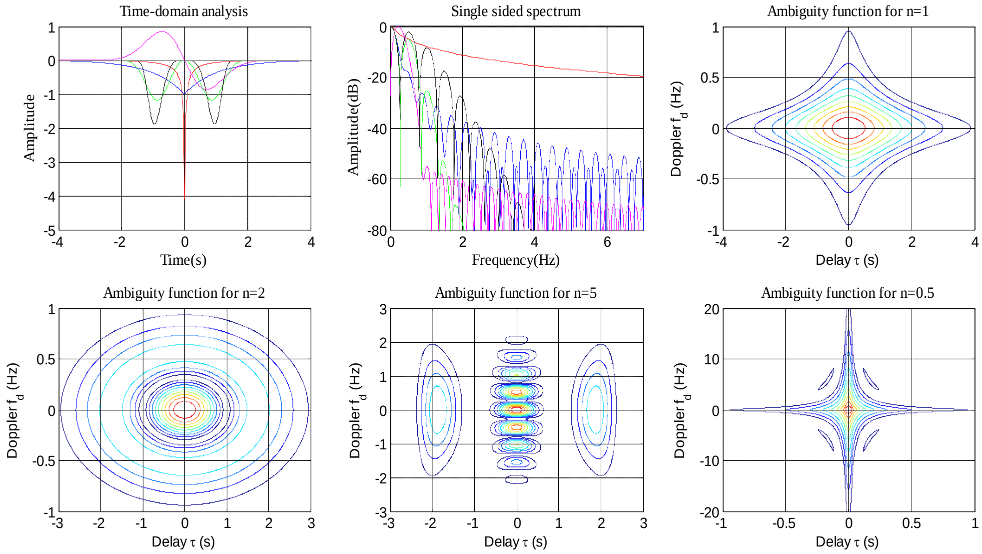

Fig.3 Time domain, frequency domain and ambiguity functions for the exponential signal ($x_3(t)$) with different exponents.

But usually for the transmissionpurposes, modulation of the input signal is required. Now these exponential pulses are to be multiplied by sinusoidal signal. Here cosine signal is incorporated having frequency 4Hz.In this paper, this frequency is taken for convenience; by altering this frequency, conclusion will not be altered, so this frequency can be up scaled or down scaled as per requirements.

By multiplying with cosine signal to the simple exponential pulses, their time domain analysis (auto correlation and matched filter response) and frequency spectrum are analyzed. Their frequency spectrum is shifted at frequency 4Hz. as represented in Fig.4 to Fig.6.

To compare the different pulses, quantitative measuring parameters such as peak side lobe level (PSL), 3-dB beam-width, and main lobe width has been compared.

Mathematically cosine exponential pulses can be represented as

$$x_4(t) = \alpha \cdot \cos(2\pi f t)\left(e^{t^n}.u(-t) + e^{-t^n}.u(t)\right) \quad (11)$$

Now multiply the bi-phase exponents to the cosine signal as represented below

$$x_5(t) = \alpha \cdot \cos(2\pi f t)\left(e^{t^n}.u(-t) - e^{-t^n}.u(t)\right) \quad (12)$$

Now multiply the differentiated exponents to the cosine signal as represented below

$$x_6(t) = \alpha \cdot \cos(2\pi f t)(n.t^{n-1}.e^{t^n}.u(-t) - n.t^{n-1}.e^{-t^n}.u(t)) \quad (13)$$

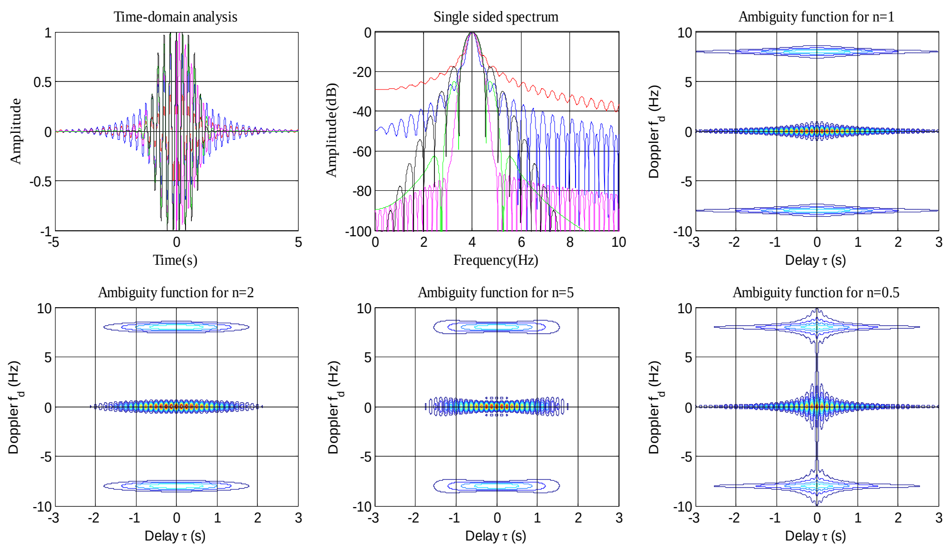

Fig.4 Time domain, frequency domain and ambiguity functions for the exponential signal ($x_4(t)$) with different exponents.

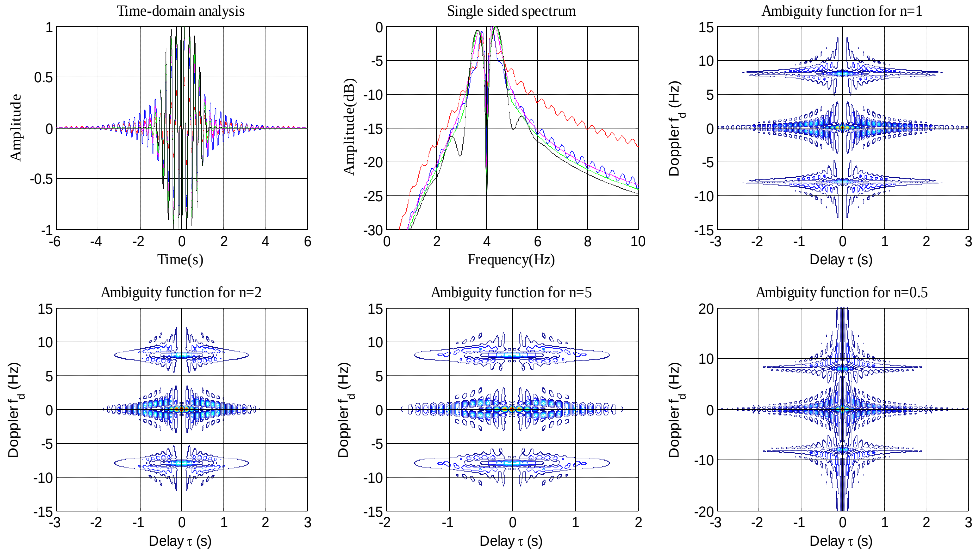

Fig.5 Time domain, frequency domain and ambiguity functions for the exponential signal ($x_5(t)$) with different exponents.

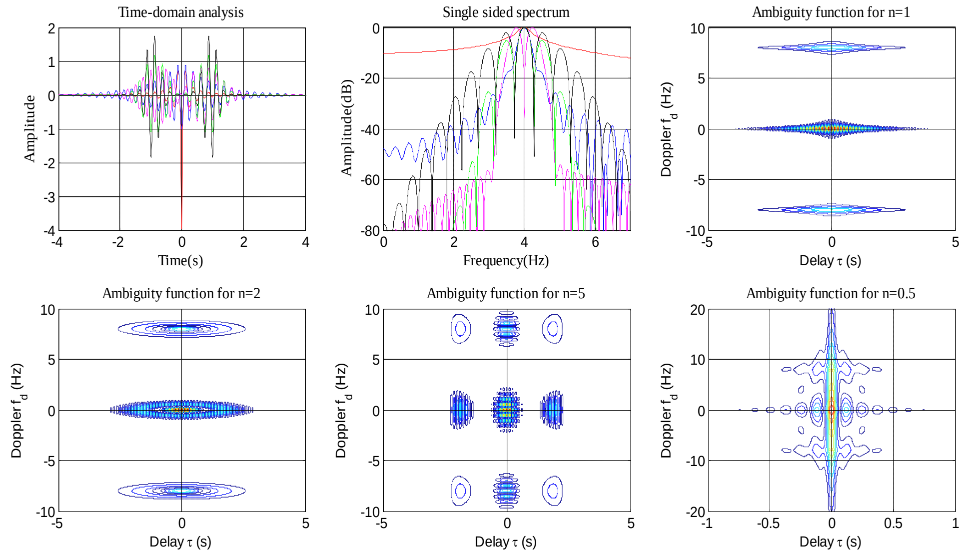

Fig.6 Time domain, frequency domain and ambiguity functions for the exponential signal ($x_6(t)$) with different exponents

Table.1 The quantitative parameters for the simple exponential pulses.

Sr. No.	Order(n)	Peak side lobe level(dB)	3dB Beam width	Main lobe width
1	0.5	-Nil-	0.14	0.70
2	1	-Nil-	0.13	0.76
3	2	-Nil-	0.19	2
4	3	-Nil-	0.21	1.12
5	5	-Nil-	0.23	1.08

Table.2The quantitative parameters for the bi-phase exponential pulses. By observing this table pulse having order half gives better PSL and order five have worst PSL.

Sr. No.	Order(n)	Peak side lobe level(dB)	3dB Beam width	Main lobe width
1	0.5	-16.19	0.34	0.86
2	1	-13.82	0.34	0.92
3	2	-11.56	0.53	--
4	3	-9.32	0.57	--
5	5	-7.69	0.59	2.2

Table.3The quantitative parameters for the differentiated exponential pulses.

Sr. No.	Order(n)	Peak side lobe level(dB)	3dB beam width	Main lobe width
1	0.5	-Nil-	0.26	--
2	1	-Nil-	0.13	0.80
3	2	-6.94	0.10	1.86
4	3	-6.03	0.14	0.56
5	5	-6.10	0.14	0.54

IV. Discussions with ambiguity functions:

In ambiguity analysis, x-axis represents the delay and y-axis represents the Doppler shift. If the same pattern is repeating in x-axis then there is an ambiguity in range resolution. If same pattern is repeating in y-axis then there is an ambiguity in the Doppler shift.In this paper the ambiguity function has been analyzed for different exponents of order one, two, five and half. For a simple exponential pulse, ambiguity function is not giving any ambiguities in range and velocities as represented in Fig.1, as there is no repetition of the pattern along x-axis and y-axis. Moreover order half has good ambiguity function as pattern is more concentrated towards the origin in comparison with other exponents.

For bi-phase exponential pulse, ambiguity function analyses is also not giving any range and velocity resolution similar to previous caseas represented in Fig.2. But fractional order have good ambiguity results because pattern is more concentrated at origin and this analysis are good than simple exponential pulses because in half order bi-phase exponent pattern come closer to origin.

Fig.3 represents the ambiguity analysis of differentiated exponential pulses. It is observed that for order one, two and half,there are no ambiguities but for order five both ambiguities exist. It is observed that for order half, ambiguity function isbetter as compared to other exponents because pattern is more concentric at the origin. So non-integer exponents gives good result for pulse compression as well as better ambiguity analysis.

The above pulses are multiplied with sinusoidal carrier of frequency 4Hz. These pulses generally represents the actual transmitted signals in practice. The ambiguity function is obtained in the similar manner for these pulses and it is observed that when frequency is increasing then delay patterns come closer to each other and Doppler shift is increasing which is repeating after twice of the applied frequency. If in the delay axis pattern come closer its range resolution is good because pulse become narrower. Here applied frequency is 4Hz hence at8Hz, 12Hz and an integer multiples of 4Hz, the same pattern is repeating on the y-axis which gives thevelocity ambiguity and same pattern is repeating on the x-axis which gives the ambiguity in range simultaneously.

Fig.5 represents the ambiguity functions for bi-phased cosine exponential pulse, which is giving ambiguities in both range and velocity.

Finally, Auto Correlation Function (ACF) whichis the output of the matched filter, represented in Fig.7 for all the above concerned cases. The best auto correlation function should have the narrow beam and very small relative side lobe levels. From Fig.7 it is observed that the output of the matched filter for non-integer value is giving better result compared to the other forms of the signals. It is observed that pulse is narrower and it has no side lobe which is desired. Order one also have not any side lobe but the matched filter response is wider which is not desired. So results are good for non-integer order pulse.

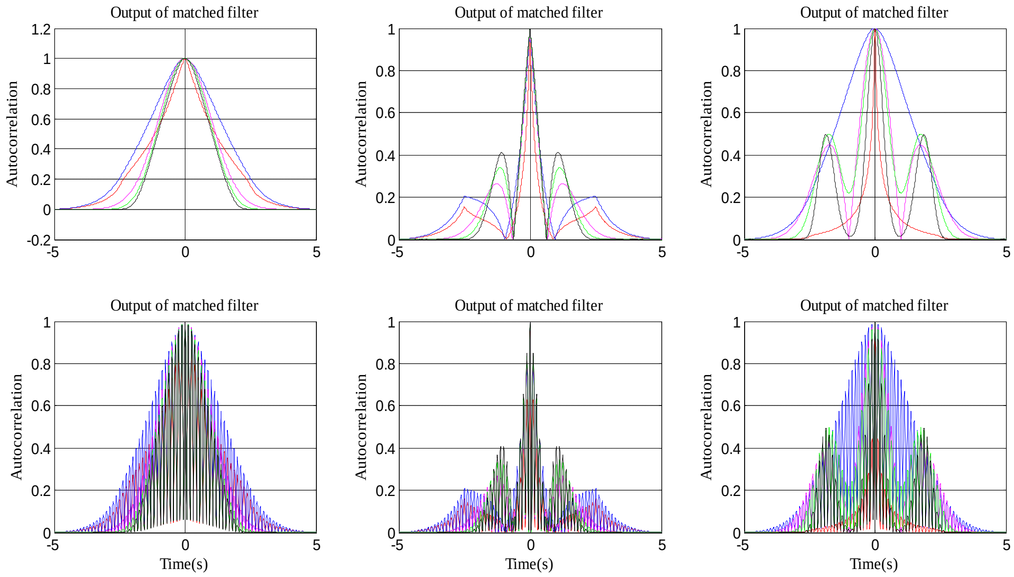

Fig.7 Autocorrelation function for signals from $x_1(t)$ to $x_6(t)$ in sequence.

V. Conclusion

In this paper a detailed analysis on the quantitative parameters of the pulse compression is carried out with an exponential kind of pulse in different mathematical forms, whose exponent can be an integer or non-integer. From the analysis, it is observed that the differentiated exponential with non-integer exponents is giving slightly better requirements with reference to pulse compression, which can be used in practical applications.

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